Mentoring model based on the levelling of probabilistic thinking to develop problem solving ability

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Abstract. This study aims to describe the probabilistic thinking level of mathematics education students in the topics of sample space, event, and the probability of the event. Many studies have produced different levels of probabilistic thinking, allowing for another new leveling structure of the probabilistic thinking.This study used a qualitative descriptive method involving 112 mathematics education students of Universitas Ahmad Dahlan who took elementary statistics subject in the second semester of 2017/2018 academic year as its subjects. The data analysis was conducted by describing the answers based on the probabilistic thinking leveling indicator.The results of the first stage of this study found a new leveling structure in probabilistic thinking, namely six levels of probabilistic thinking: (1) Level 0 (pre-subjective), with 23 students (20.54%); (2) Level 1 (subjective), with 18 students (16.07%); (3) Level 2 (transition), with 16 students (14.29%); (4) Level 3 (informal quantitative), with 17 students (15.18%); (5) Level 4 (formal quantitative), with 14 students (12.50%); and (6) Level 5 (numerical), with 24 students (21.43%). The second phase of the research will focus on the appropriate mentoring model based on the findings of this first stage research, improve the students' ability to solve problems, and raise their level of probabilistic thinking to the next levels.

1. Introduction

Researches related to probabilistic thinking were carried out by researchers starting in the 1990s. Then, in the period of 2000s to 2010s, researchers were concerned that this period was called the contemporary research period, while the next ongoing period is the assimilation period [1].Several studies have been conducted by researchers. For example, researches conducted by Jonesand Polaki found four probabilistic thinking levels, namely Level 1 (subjective), Level 2 (transitional), Level 3 (informal quantitative, and Level 4 (numerical) [2–4]. Sujadi also developed the level offered by Jones and added one more level of probabilistic thinking for junior high school students who had not been given probability material, concluding that junior high school students who had not formally learned about probability had a gap in probabilistic thinking [5]. Concrete object manipulation, as well as computer animation, can help students improve student schemes which are at Level 0(pre-subjective probabilistic thinking) [6]. It is related to the understanding of possible outcomes and reduces the influence of irrelevant aspects, even though at the end of the learning, the students still could not register the members of the sample space from a one-level experiment.

Nacarato and Grando have four stages in building probabilistic thinking, namely: (1) the classical concept phase, (2) the frequency or empirical concept phase, (3) the subjectivist concept phase, and (4) the axiomatic or formal concept phase [7]. Borovcnik has four stages in establishing probabilistic

thinking, namely: (1) probability as index of surprise stage, (2) feedback from probabilistic situations is indirect, (3) causal alternative to randomness, and (4) non-probabilistic criteria for decisions [8,9].

There are many levelling structures in probabilistic thinking developed by the researchers. Thus, it is very possible that there is another levelling structure in probabilistic thinking at the student level. Probabilistic thinking includes higher order thinking skills. It can be seen from Jean Piaget's classification in the main characterization of one's cognitive development, that logic and probability are positioned at the stage of formal operations [10].

In solving probabilistic problems, many students use strategies based on beliefs, previous experiences (in daily activities and at school), and intuitive strategies [11]. Four rubric categories have been identified which can be considered to describe how students build meaning for probability questions. While students demonstrate competency with theoretical interpretations, they are less competent on tasks that involve definitions of probability. It is because they ignore learning points of view from linguistic problems. The probability must be understood as a kind of cultural knowledge created by all cultures. It may not look the same from one cultural setting to another. With so many ethnic groups in the main class throughout the world with a frame of probability program, it's important to really listen to students' voices to understand what or what might not be suitable for these students in terms of their probabilistic thinking. Teachers, curriculum developers, and researchers need to work together to find better ways to help all students in developing their probabilistic thinking [12].

Furthermore, the probabilistic reasoning for male junior high school students in problem solving related to the probability of an event tends to appropriately respond to various situations in an uncertainty context. Their reasoning in each step of problem solving was good as it was shown by quickly deciding the strategies that will be used in solving the problems given [13].

The next development of research related to the probabilistic thinking is about probabilistic models. The meaning of model is viewed from two perspectives. The first is a mathematical model as a decision-making tool. The second is the way of thinking or representing an idea. Thus, in a simple and concrete incarnation, a model is a representation of another object. Furthermore, the term "model" is used to describe the way people understand their world. Meanwhile, another view states that mathematical modelling competence of real phenomena is a must for all components of mathematical literacy, and is needed in public education both now and in the future [14].There are three approaches of modeling situations with randomness, the classical approach, the frequentistic approach, and the subjectivistic approach [15]. There are also three interconnected ways of thinking about probability, the true probability, the model probability, and the empirical probability [16]. Moreover, theories and models that have been developed for certain probabilistic concepts and processes can be synthesized. This model shows a different level or pattern of growth in probabilistic reasoning, the findings of the role of modeling become recommendations for policies in curriculum development [17].

Based on the observation we have done in the probability theory subject, we found that permutation, combination, and conditional probability was the materials which we have to focus on. When students are faced with problems, they often make mistakes to distinguish permutation and combination problems. The previous study found that the results obtained from the test sheets and interviews show that the thinking of the ninth-grade students of SMA 9 Bengkulu in solving the permutation and combination was still at the level of the transitional thinking (Level 2) [18]. In the other study, the level of probabilistic thinking of Group 1 and 2 were at Level 2, with percentages respectively 71.89% and 44.44% [19].

The views of the experts and the results of the research are very interesting and need to be developed and followed up at the college level, because knowing the potential of students will greatly assist the lecturer in delivering the material, both in terms of the learning approach or the content, as well as guidance and attention to them.All types of thinking including thinking of probabilities are closely related to one's efforts to find solutions to the problems they face. The solutions that are obtained by each person will vary, both in terms of quality, accuracy, and speed. The difference in results is related to the level of thinking of the probabilities of each individual, and it is very possible that the level of thinking of the probability of a student will influence the solution of the problem solving.

Based on the description, we were interested in conducting research related to the mentoring model based on probabilistic thinking level to develop students' abilities in problem solving. This research was carried out in two stages, the first stage was carried out to find the level structure in probabilistic thinking. The next stage is to provide assistance on the basis of findings in the first stage. With the research in the second stage, it is expected to find a mentoring model that matches the probabilistic thinking level of students in problem solving, so that it will raise the level of their upper levels of probabilistic thinking.

2. Methods
2.1. Research design
In this study, we measured the ability of students related to problem solving probabilities, namely: permutation, combination, sample space, events, and the probability of the events. From the analysis of the data obtained, it will produce an initial portrait of the student's probabilistic thinking process, which is grouped according to level. Then, we conducted interviews with the subjects who met the criteria to reveal the probabilistic thinking process.

2.2. Participants
The subjects in this study were 112 students of mathematics education department of Universitas Ahmad Dahlan in the second semester of the 2017/2018 academic year who took theelementary statistics course, selected to meet the criteria for each level in probabilistic thinking.

3. Results and discussion
We administered the probability problems to all participants and the results show that they have different levels of probabilistic thinking. The most interesting thing was that there are participants who cannot be included in any level of probabilistic thinking developed by previous researchers,especially by Jones. There were some students who could not register members of the sample space as they did not understand the problems. They can be grouped at the level below the subjective level. We could name it the pre-subjective level or the Level 0. Another finding is that there was a group which could not be categorized in any levels developed by Jones. They used their strategies correctly, but the results were not complete, or they did not use strategies, but the results were complete. They have to be grouped into separate groups between the informal quantitative level and the numeric level with the term formal quantitative (Level 4). Thus, in this study, there are six probabilistic thinking levels with the indicators presented in Table 1.

Table 1. Indicator of the probabilistic thinking level in the material of the sample space and the probability of an event.

Level	Indicator
0-Pre-subjective	- Could not understand the problem. - Could not register the members of a sample space.
1-Subjective	- Do not choose any problem solving strategy. - The solution is not complete and false. - Do not provide the probability.
2-Transition	- Do not use any strategy. - The solution is correct but not complete. - Do not provide the probability.
3-Informal quantitative	- Use a strategy but is not correct. - Consistently use the strategy. - The solution is almost complete. - Do not provide the probability.

| 4-Formal quantitative | - Use a correct strategy. |
	- Consistently use the strategy.
	- Provide an incomplete solution.
	or
	- Do not use strategy.
	- The solution is correct.
	- Do not provide the probability.
5-Numeric	- Use a correct strategy.
	- Consistent
	- The result is complete and correct.
	- Provide the probability.

Table 1 shows a variety of probabilistic students' thinking levels. The number of levels found in this study is 6 levels of probabilistic thinking, indicating that there are other structures in the student's probabilistic thinking level. Jones found 4 levels in probabilistic thinking and Sujadi found 5 levels in probabilistic thinking. It is interesting that we can add Level 4 formal quantitative in the structure[2,5].

The number and percentage of 112 students for each level of probabilistic thinking can be seen in Table 2. The least number of students is at Level4 (formal quantitative) with 14 students and the percentage of 12.50%, while level-0 and level -5 balanced with the number and percentage of each respectively 23 students (20.54%) and 24 students (21.43%), while the other 3 levels were relatively the same.

Table 2. Indicator of the probabilistic thinking level in the material of the sample space and the probability of an event.

Level	Number	Percentage
Level 0	23	20.54
Level 1	18	16.07
Level 2	16	14.29
Level 3	17	15.18
Level 4	14	12.50
Level 5	24	21.43
Sum	112	100.

Level-4 with a total of 14 students or a percentage of 12.50% can also be said to be the transition level from the informal quantitative (Level 3) to the numeric (Level 5), because this Level 4's indicator does not match the previous level and not the next level, either. We can see in Figure 1.

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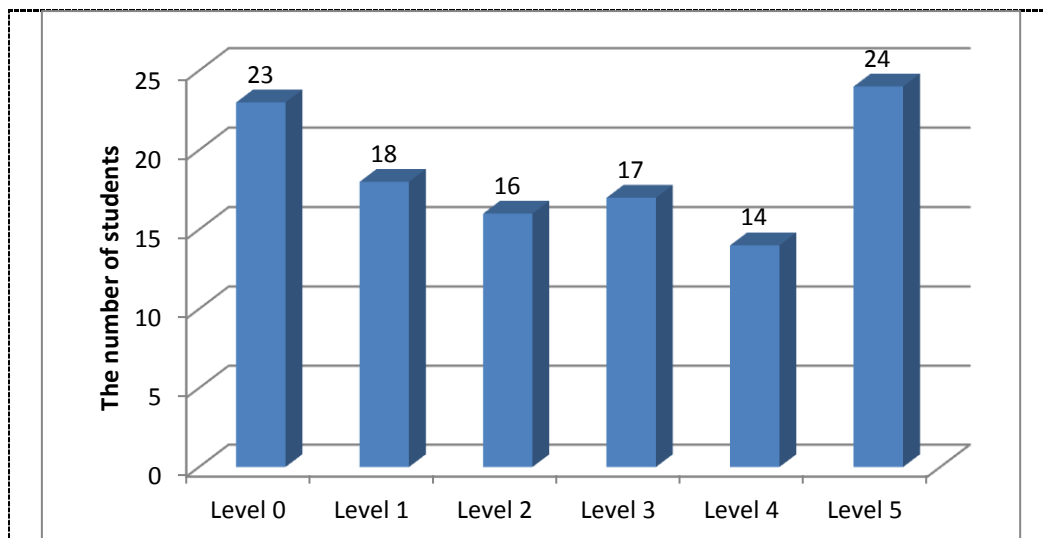
Figure 1. The number of students categorized in each level of probabilistic thinking.

The findings of this level of probabilistic thinking, are the results of the first stage of the research. They will be followed by the second stage of research which focuses on the discovery of mentoring models on the findings of this first stage research in improving students' ability to solve problems.

4. Conclusion

There are many types of levelling in the students' probabilistic thinking. It shows the level of students' ability to solve problems. In this study, a new structure was found in the probabilistic thinking level of mathematics education students, namely Level 0 (pre-subjective), Level 1 (subjective), Level 2 (transitional), Level 3 (informal quantitative), Level 4 (formal quantitative), and Level 5 (numerical).Level-4 with a total of 14 students or a percentage of 12.50% can also be said to be the transition level from the informal quantitative (Level 3) to the numeric (Level 5), because this Level 4's indicator does not match the previous level and not the next level, either.

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